MATH 147: THINGS TO KNOW AND PRACTICE PROBLEMS FOR EXAM 1

I. For Exam 1 you will need to:

(i) Know the definition of the limit of a function of two variables, know how to calculate such limits or determine such limits do not exist and know the relevance of limits to the concept of continuity.
(ii) Know limit definitions of partial derivatives and how to calculate a partial derivative using a limit.
(iii) Use the limit definition to determine if a function is differentiable.
(iv) Know how to calculate first and second order partial derivatives.
(v) Know the important theorems from class concerning continuity, differentiability, and equality of mixed partials.
(vi) Know how to: calculate partial derivatives, use and verify the chain rule, calculate the tangent plane, calculate the gradient (and know what the gradient means), calculate the directional derivative.
(vii) Know how to find critical points of $f(x, y)$ and how use the second derivative test to classify the critical points.

II. Practice problems.

1. For the function $f(x, y) = 3x^3y^2 + 4xy - 7x$:
(i) Find the tangent plane to the graph of $z = f(x, y)$ at (1,2).
(ii) Verify that the gradient of $F(x, y, z) = z - f(x, y)$ at (1,2) is normal to the plane in (i).
(iii) Find $D_{\vec{u}}f(1, 2)$ for $\vec{u}$ the unit vector in the direction of $3\vec{i} + 2\vec{j}$ and also in the direction of $\nabla f(1,2)$.

2. Calculate $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ both by substitution and by using the chain rule for the function $f(x, y, z) = xy^2z^3$ with $x = u^2 + v$, $y = 3v + 7$, and $z = 3u^3$.

3. Evaluate the following limits or show they do not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^4 + x^2y^2 + y^4} \qquad \lim_{(x,y)\to(2,1)} \frac{x^4 \cos(\pi y)}{e^{x+y}} \qquad \lim_{(x,y)\to(0,0)} \frac{|x|}{|x| + |y|} \qquad \lim_{(x,y)\to(0,0)} \frac{x^3 + y^3}{x^2 + y^2}$$

4. Consider the function $f(x, y) = \begin{cases} 0 & \text{if } xy \neq 0 \\ 1 & \text{if } xy = 0 \end{cases}$. Show that $f(x, y)$ is not continuous at (0,0), then show that $\frac{\partial f}{\partial x}$(0,0) and $\frac{\partial f}{\partial y}$(0,0) exist. Why does this not contradict the theorem which states that if $f(x, y)$ is differentiable at (a, b), then $f(x, y)$ is continuous at (a, b)?

5. For the function $f(x, y) = 3x^2 + 7y - 2$, use the limit definitions: (a) To verify that $f(x, y)$ is differentiable at (3,2) and (b) To verify that the first order partials of $f(x, y)$ are continuous at (3,2).

6. Use the limit definition to calculate the directional derivative of of $f(x, y) = 2x^2y - 3x$ at (4,3) in the direct of the vector $\vec{i} + \vec{j}$. Verify your answer by dotting the gradient vector with an appropriate direction vector.

7. For the function $f(x, y) = \begin{cases} \frac{x^3}{x^2+y^2}, & \text{if } (x, y) = (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$, show that $D_{\vec{u}}f(0, 0)$ exists for all directions $\vec{u}$, but $f(x, y)$ is not differentiable at $(0, 0)$.

8. Find and classify the critical points for: $f(x, y) = 2x^2 - 4xy + y^4 + 2$ and $g(x, y) = x^3 - 12x + y^3 + 3y^2 - 9y$.

9. Show that the surface area of a closed rectangular box with volume 27 in^3 is smallest when the box takes the shape of a cube.

10. Show that the sum of the squares of the distances from a point $P = (c, d)$ to n fixed points $(a_1, b_1), \ldots, (a_n, b_n)$ is minimized when c is the average of the x-coordinates a_i and d is the average of the y-coordinates b_i.